ed the hypothesis either of right angle or obtuse angle, and from the other the hypothesis of acute angle; which is contrary to P.V, P.VI, and P.VII.

Therefore the three angles together of either of the aforesaid triangles will be less than two right angles; and therefore (P.IX.) is established the hypothesis of acute angle. Quod erat tertio loco demonstrandum.

Accordingly by any triangle ABC, of which the three angles are equal to, or greater, or less than two right angles, is established respectively the hypothesis of right angle, or obtuse angle, or acute angle. Quod erat propositum.

COROLLARY. Hence, any one side of any proposed triangle being produced, as suppose AB to H, the external angle HBC will be (Eu.I.13.) either equal to, or less, or greater than the remaining internal and opposite angles together at the points A and C, according as is true the hypothesis of right angle, or obtuse angle, or acute angle. And inversely.

[To be continued.]

THE "IRREDUCIBLE CASE."

By J. K. ELLWOOD, A. M., Principal Colfax School, Pittsburg, Pa.

PROBLEM.—*To extract the cube root of* $a\pm\sqrt{-b}$.

Put $\sqrt[3]{a+\sqrt{-b}}=m+n$, and $\sqrt[3]{a-\sqrt{-b}}=m-n$.

Then $a+\sqrt{-b}=m^3+3m^2n+3mn^2+n^3$, and $a-\sqrt{-b}=m^3-3m^2n+3mn^2-n^3$. Hence $a=m^3+3mn^2$, and $\sqrt{-b}=3m^2n+n^3$.

Example 1. Find the cube root of $9+25\sqrt{-2}$.

Here $a=m^3+3mn^2=9=3^3-18$. Hence $3mn^2=-18$, and $n=\sqrt{-2}$.

To verify these values of m and n substitute them in $\sqrt{-b}=3m^2n+n^3=25\sqrt{-2}$. Doing this we have $27\sqrt{-2}+(-2\sqrt{-2})=25\sqrt{-2}$.

$\therefore$ $3+\sqrt{-2}$ is the required root. When the substituted values of m and n do not give the second term they are not correct, and other values must be found by trial.

Example 2. Find the cube root of $2\sqrt{11}+30\sqrt{-3}$. Here $a=m^3+3mn^2=2\sqrt{11}=(\sqrt{11})^3-9\sqrt{11}$.

Hence $3mn^2=-9\sqrt{11}$, and $n=\sqrt{-3}$. Since these values of m and n substituted in $3m^2n+n^3$ give $\sqrt{-b}=30\sqrt{-3}$, the root is $\sqrt{11}+\sqrt{-3}$.

This method frequently enables us to simplify Cardan's formula for cubics in what is called the "irreducible case", said formula being

$$x=\sqrt[3]{q+\sqrt{(q^2-p^3)}}+\sqrt[3]{q-\sqrt{(q^2-p^3)}}.$$

1. * $x^3-22x-24=0$. Here $p=\frac{22}{3}$, $q=12$.

$$\text{Then } x=\sqrt[3]{12+\tfrac{26}{3}\sqrt{(-\tfrac{10}{3})}}+\sqrt[3]{12-\tfrac{26}{3}\sqrt{(-\tfrac{10}{3})}}.$$

Now, $a=m^3+3mn^2=12=-2^3+20$. $3mn^2=20$, $n=\sqrt{-\frac{10}{3}}$.

$\therefore$ $-2\pm\sqrt{-\frac{10}{3}}$ are the roots. Then $x=(-2+\sqrt{-\frac{10}{3}})+(-2-\sqrt{-\frac{10}{3}})=-4$.

2. $x^3-8x^2+19x-12=0$. Put $x=y+\frac{8}{3}$.

Then $y^3-\frac{7}{3}y+\frac{20}{27}=0$, where $p=\frac{7}{9}$, $q=-\frac{10}{27}$.

$$\text{Then } y=\sqrt[3]{-\tfrac{10}{27}+\sqrt{(-\tfrac{243}{729})}}+\sqrt[3]{-\tfrac{10}{27}-\sqrt{(-\tfrac{243}{729})}}$$

$$=\sqrt[3]{-\tfrac{10}{27}+\tfrac{1}{3}\sqrt{-3}}+\sqrt[3]{-\tfrac{10}{27}-\tfrac{1}{3}\sqrt{-3}}$$

Here $a=m^3+3mn^2=-\frac{10}{27}=(\frac{2}{3})^3-\frac{18}{27}$. Hence $n=\sqrt{-\frac{1}{3}}=\frac{1}{3}\sqrt{-3}$.

$\therefore$ $y=(\frac{2}{3}+\frac{1}{3}\sqrt{-3})+(\frac{2}{3}-\frac{1}{3}\sqrt{-3})=\frac{4}{3}$. Then $x=\frac{4}{3}+\frac{8}{3}=4$.

3. $x^3-12x^2+41x-42=0$. Put $x=y+4$.

Then $y^3-7y=6$, where $p=\frac{7}{3}$, $q=3$. Then $y=\sqrt[3]{3+\frac{10}{9}\sqrt{-3}}+\sqrt[3]{3-\frac{10}{9}\sqrt{-3}}$.

Here $a=3=(\frac{3}{2})^3-\frac{3}{8}$. $\therefore$ $n=\frac{1}{6}\sqrt{-3}$.

$\therefore$ $y=(\frac{3}{2}+\frac{1}{6}\sqrt{-3})+(\frac{3}{2}-\frac{1}{6}\sqrt{-3})=3$, and $x=3+4=7$.

* Maynard in his Key to Bonnycastle's Introduction, page 78, says this "can only be resolved by a table of sines, or by infinite series."

THEOREM 16 OF LOBATSCHEWSKY'S THEORY OF PARALLELS.

By JOHN N. LYLE, Ph. D., Professor of Natural Sciences, Westminster College, Fulton, Missouri

Says Lobatschewsky in his Theorem 16—"All straight lines which in a plane go out from a point can, with reference to a given straight line in the same plane, be divided into two classes—into *cutting* and *not-cutting*. The *boundary lines* of the one and the other class of those lines will be called *parallel to the given line.*

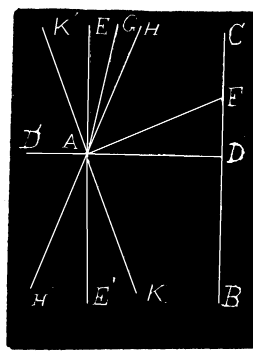

Fig. 1.

From the point A (Fig. 1) let fall upon the line BC the perpendicular AD, to which again draw the perpendicular AE. In the right angle EAD either will all all straight lines which go out from the point A meet the line DC, as for example AF, or some of them, like the perpendicular AE, will not meet the line DC. In the uncertainty whether the perpendicular AE is the only line which does not meet DC, we will assume it may be possible that there are still other lines, for